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Technical note

On the use of late-time drawdown in interpreting aquifer pumping test

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The review aims to provide a common understanding of the use of late-time drawdown to interpret aquifer pumping tests. The first part of the review provides an overview of the use of the late-time drawdown in literature to illustrate where and how the term is being used. A discussion on the practical implications of using the term and its significance is then presented. The review shows the use of the late-time drawdown in three main ways: the application of the Cooper and Jacob time-drawdown method, the description of the third segment of the unconfined aquifer drawdown-time curve, and when trying to estimate representative/effective transmissivity parameters in heterogeneous aquifers. Unlike the other two situations, the use of late-time data in typical unconfined aquifers is supported by the groundwater flow principles and hence has a meaningful application. The aspects highlighted in this review are important to improve the theoretical and practical knowledge required for analysing and interpreting aquifer pumping test data.

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INTRODUCTION

Over the past 10 decades, late-time drawdown has featured consistently in the groundwater science literature of aquifer pumping tests analysis (Tóth, 1966; Neuman, 1972; Lachassagne, 1989; Butler, 1991; Copty, 2004; Tumlinson, 2006; Neuman et al., 2007; Çimen, 2009; Pechstein et al., 2016; Kahuda and Pech, 2020). However, a review of the use of the late-time drawdown in pumping test analysis is yet to been done. This has resulted in it being used without discretion and in some cases inappropriately. In simple terms, one can define 'late-time drawdown' as the drawdown towards the end of an aquifer pumping test activity. This implies that there must be a clearly defined end for any aquifer pumping test activity. With this definition in mind, a few aspects which need explanation and rationalization for the meaningful and generalized practical application of the late-time drawdown need to be highlighted.

Firstly, it is the appropriate duration of an aquifer pumping test activity which can enable the observation of this late-time drawdown data phenomenon during a pumping test. Secondly, how late should it be during that pumping test duration for the late-time drawdown phenomenon to occur? This is very important because it involves time, which is also a very relative parameter. Thirdly, it is also not clear how to assess and evaluate the appropriateness of late-time drawdown data for parameter estimation. These are some of the aspects which make the practical use of the term very challenging.

This paper provides a technical review of the use of the late-time drawdown, which is of paramount importance to provide clarity among groundwater practitioners who are normally faced with the very daunting task of frequently interpreting aquifer pumping test data. The work complements recent studies aimed at improving the practical understanding of the classical methods of analysing aquifer pumping test data. Flores and Bailey (2019) revisited the Theis solution derivation to enhance its understanding and application. Gomo (2019 and 2020) illustrated the use of infinite acting radial flow (IARF) as an objective criterion to determine the applicability of the Cooper and Jacob time-drawdown (1946) and distance-drawdown methods. Qiwen et al. (2022) recently showed that the generalized radial flow model better characterises groundwater flow regimes of heterogeneous pore aquifers from pumping test data. The review is motivated by the need to continuously improve the theoretical and practical knowledge of analysing aquifer pumping test data among groundwater practitioners.

The first component of this paper gives an outline to illustrate how the term is being used in literature and is followed by a discussion of the implications of the use of the term and an evaluation of its practical significance.

Use of late-time drawdown in literature

This section is aimed at showing the where and how the term late-time drawdown data has been used in literature. The author does not claim to have covered all uses of the term in literature but does believe the examples given will help to improve the understanding and create a basis for further discussion and knowledge improvement.

The use of late-time drawdown data in aquifer pumping test analysis can be tracked back to Tóth (1966). In this study, Tóth (1966) indicated to have used late-time drawdown data on the semi-log plots to estimate well yield in typically heterogeneous aquifers. Further along the years, Neuman (1972) developed the theory of groundwater flow in an unconfined aquifer under the influence of gravity where the time-drawdown curve is divided into three segments: early-time, middle-time, and late-time. The late-time drawdown where the flow becomes entirely horizontal and water is released from storage only by gravity drainage converge on the Theis solution. Studies such as Lachassagne et al. (1989) and Butler (1991) suggested that the transmissivity parameter estimated from late-time drawdown data through the use of classical methods such as the Cooper and Jacob (1946) time-drawdown method approximate the geometric mean of the regional transmissivity. The basis for this suggestion was that after a relatively long time of pumping the regional groundwater system would be influencing the flow towards a pumping well, hence the use of late-time drawdown data. However, the limitation of this lies in defining what is a relatively long time pumping time.

Van Tonder et al. (2000) use the terms early-time and late-time transmissivity calculated using drawdown derivatives (Eq. 1) (Spane, 1963; Renard, 2008) for estimating borehole sustainable yield. The early-time transmissivity is calculated using the derivative of drawdown at radial flow while the late-time transmissivity is calculated using the maximum derivative of drawdown. The maximum derivative of drawdown occurs at the end of a pumping test. The late-time transmissivity therefore reflects the last observed flow conditions during a test and as such can be used to extrapolate the drawdown to estimate the borehole sustainable yield.

$$\frac{\partial s}{\partial \log t} = 2.3 \frac{Q}{4\pi T} \tag{1}$$

where s is the drawdown measured in a pumping well (L); Q is the constant pumping rate (L^3/T); T is aquifer transmissivity (L^2/T) and t is the time for which the transmissivity is calculated (T).

Copty and Findikakis (2004) concluded that the transmissivity estimates made using the Cooper and Jacob (1946) late-time drawdown data represent the geometric mean of the parameter for a heterogeneous system. Tumlinson et al. (2006) concluded that using the method of Cooper and Jacob (1946) to interpret late-time drawdown data from a pumping well in a heterogeneous aquifer resulted in large-scale representative transmissivity. On the contrary, Neuman (2007) suggested that the use of the Cooper and Jacob (1946) method to estimate transmissivities using late-time drawdown does not provide good estimates of geometric mean transmissivity of heterogeneous aquifers.

Later, Çimen (2008) proposed a procedure based on the principles of the Cooper and Jacob (1946) time-drawdown method to estimate aquifer parameters using the late-time drawdown data in multi-well pumping tests performed in confined aquifers. Pechstein et al. (2016) suggested that the Cooper and Jacob (1946) method and the continuous-derivation method estimate equal transmissivity and storativity when applied to analyse late-time drawdown data. Recently, Kahuda and Pech (2020) developed a method for determining the need for well rehabilitation using the early-time aquifer pumping test data. This approach they suggested can be utilised when the late-time data are missing, making it impossible to apply the Cooper and Jacob (1946) method. In proposing a methodology for the interpretation of aquifer tests in relation to CO_2 residual trapping experiments, Martinez-Landa et al. (2021) concluded that the late-time drawdown data is non-unique, but its analysis with different models did not significantly affect the estimation of trapped CO_2. A summary of the use of late-time drawdown in literature is presented in Table 1.

DISCUSSION

The author does not claim that the literature outline provided above is exhaustive, but it does give a good reflection of how the late-time drawdown has been used in various ways from 1966 to 2021. From the literature review, three main aspects about the use of late-time drawdown can be summarised and discussed:

- Late-time drawdown data use with the Cooper and Jacob (1946) time-drawdown method to analyse the aquifer pumping test data
- Use in description of the third segment of the unconfined aquifer drawdown-time curve
- Use in attempting to estimate representative/effective transmissivity parameters in heterogeneous aquifers

The late-time drawdown has been used when the drawdown data are plotted on a semi-log graph of drawdown against time, typically referred as the 'Cooper and Jacob plot'. The use of the Cooper and Jacob (1946) method is traditionally evaluated on basis the of u values (Eq. 2), which become smaller as the pumping time increases.

$$u = \frac{r^2 S}{4Tt} \tag{2}$$

where: T is transmissivity of the aquifer (L^2/day), r is distance of the observation well from the pumping well (L), S is storativity of the aquifer (no unit), and t is duration of the pumping (L).

The guidance from literature provides different threshold values of u making it difficult to objectively evaluate the method's applicability (Gomo, 2019), but it is a fact that u values tend to be smaller after a long pumping time. This probably brought the notion that the Cooper and Jacob (1946) method can generally be applicable at late-time drawdown, yet this is not always the case. Recently, Gomo (2019, 2020) illustrated how the IARF condition (Spane, 1993 and Renard et al. 2008) is a more objective evaluation criterion for the application of the Cooper and Jacob (1946) method.

Table 1. Summary of the use of late-time drawdown in literature

Author and year of publication	Title of paper
Tóth (1966)	Groundwater, geology, movement, chemistry and resources, near Olds, Alberta
Neuman (1972)	Theory of flow in unconfined aquifers considering delayed response of the water table
Lachassagne et al. (1989)	Evaluation of hydrogeological parameters heterogeneous porous media
Butler (1991)	A stochastic analysis of pumping tests in laterally non-uniform media
Van Tonder et al. (2000)	Estimation of the sustainable yields of boreholes in fractured rock formations
Copty and Findikakis (2004)	Stochastic analysis of pumping test drawdown data in heterogeneous geologic formations
Tumlinson et al. (2006)	Numerical evaluation of pumping well transmissivity estimates in laterally heterogeneous formations
Çimen (2008)	Effective procedure for determination of aquifer parameters from late time-drawdown data
Pechstein et al. (2016)	Estimating transmissivity from single-well pumping tests in heterogeneous aquifers
Kahuda and Pech (2020)	A new method for the evaluation of well rehabilitation from the early portion of a pumping test
Martinez-Landa et al. (2021)	Application to CO_2 residual trapping experiments at the Heletz site.

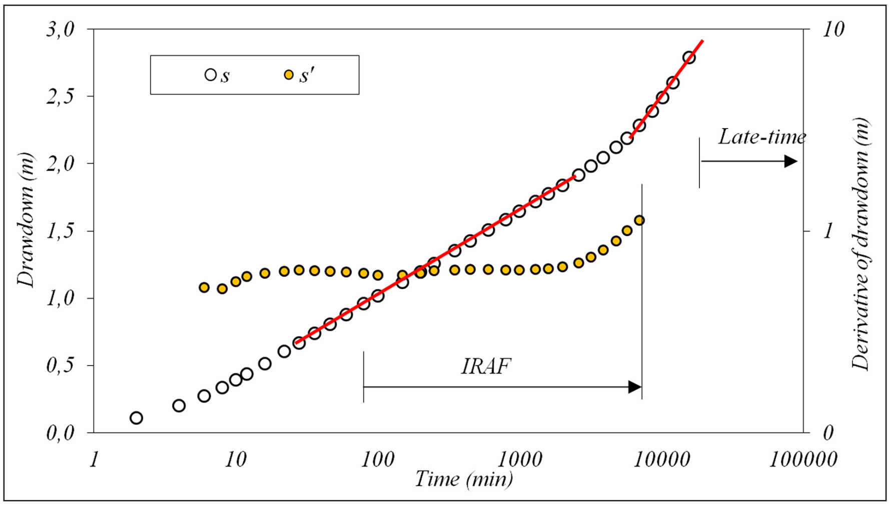

Figure 1. Graph showing drawdown (*s*) on primary vertical axis against time on a semi-log axis and derivative of drawdown (*s*′) on secondary axis against time on a log-log axis

The Cooper and Jacob (1946) method should be applied to the drawdown data under the influence of IARF conditions and these do not necessarily apply to the late-time drawdown data. It is also possible to have IARF conditions followed by other flow regimes, such as a low permeable boundary influencing the late-time drawdown data. Figure 1 illustrates an example where a low permeable boundary influences the late-time drawdown after the IARF.

The occurrence of IARF can be identified when derivative values of drawdown become constant (Spane, 1993; Renard et al., 2008; Gomo, 2019; Gomo, 2020) and the Cooper and Jacob (1946) method is applicable during the RAF period. In this example, IARF occurs before the late-time drawdown which is then affected by the low permeable boundary. With this case, estimating parameters on the basis of the late-time drawdown data using the Cooper and Jacob (1946) method would principally give incorrect results. While the Cooper and Jacob (1946) straight line can perfectly fit this late-time drawdown data, the meaning of the results is not supported by the principles underlying the method.

In contrast, the use of the late-time drawdown in a typical unconfined aquifer to describe the last segment of the drawdown-time curve (Neuman, 1972) is objective and rationalized. The typical drawdown curve for the unconfined aquifer is clearly defined and the meaning of each segment is explained. It is therefore clear how the late-time drawdown should look and the circumstances under which it occurs in a typical unconfined aquifer. This makes it possible to use the late-time drawdown data meaningfully and with the same basis of understanding across groundwater practitioners.

Another important aspect to discuss is that late-time drawdown has been used when estimating representative/effective transmissivity from aquifer pumping tests conducted in heterogeneous aquifers. Logically, estimating transmissivity with the late-time drawdown data would seem to make sense because it is reflective of effective transmissivity for the whole system, considering that the cone of depression would have passed through different formations up to the late time. But again, late-time is relative in that it is bound to change with duration of the pumping test activity, hence the meaning of what the representative transmissivity is would also vary. From the literature outline, the late-time drawdown in heterogeneous aquifers appears to have been analysed with the Cooper and Jacob (1946) method, implying that it was forming a straight-line at late-time. It does appear that the use of the late-time drawdown data is an attempt to justify the use of the Cooper and Jacob method to estimate the transmissivity in heterogeneous aquifers. It is well known that the Cooper and Jacob (1946) method is not applicable for heterogeneous aquifers and the presence of data matching a straight line at late-time seems to have been used as the basis for using the method. In cases where the late-time drawdown data does not form a straight-line, then the Cooper and Jacob (1946) method could not be used and for that reason the late-time drawdown data were not used in those instances. However, as illustrated by Gomo (2020), the applicability of the Cooper and Jacob (1946) method should be objectively evaluated based on the IARF and not the occurrence of data in a straight line. Neither does the occurrence of a straight line indicate acceptable u-values. In a practical sense, this will depend on the nature of heterogeneities and the characteristics of hydraulic features influencing drawdown at late-time. For example, the late-time data could be affected by a limiting flow boundary, more permeable formation or even impermeable formation, as determined by the nature of field heterogeneities. These hydraulic features can induce different flow characteristics on late-time drawdown data which would make it difficult to analyse it with the Cooper and Jacob (1946) distance-drawdown methods or any other method.

CONCLUSIONS

The article provided a technical review of the use of the late-time drawdown to analyse and interpret aquifer pumping test data. The late-time drawdown is mainly used in three ways:

- Application of the Cooper and Jacob (1946) time-drawdown method
- Description of the third segment of the unconfined aquifer drawdown-time curve
- Estimation of the representative/effective transmissivity parameters in heterogeneous aquifers

In the application of the Cooper and Jacob (1946) method, the use of the late-time drawdown does not give an assurance of the validity of using the method, because the late-time data are not always characterised by infinite radial acting flow conditions. The use of late-time data in typical unconfined aquifers is supported by the groundwater flow principles developed by Neuman (1972), hence has a meaningful application. The meaning of aquifer parameters estimated in heterogeneous aquifers using the late-time drawdown remains unclear.

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